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GENERALIZATION OF THE H_p-THEOREM IN A SPACE OF CONSTANT CURVATURE

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Let $x: M^n \to \mathbf{R}^{n+1}$ be and isometric immersion of an oriented Riemannian manifold M^n with unit normal vector ν, mean curvature H and support function $p = -\langle x, \nu\rangle$. The H_p-Theorem says that if M^n is compact and

$$Hp = 1,$$

then $x(M^n)$ is a round sphere ([3]).

Here we announce two generalizations of the H_p-Theorem. The proofs will appear elsewhere.

Denote by Q_c^{n+1} an n-dimensional simply connected space of constant curvature c. If $p_o \in Q_c^{n+1}$ we denote $r(\cdot) = d(\cdot, p_o)$ the distance function relative to p_o and we write grad r for the gradient of r in Q_c^{n+1}. Let $x: M^n \to \mathbf{R}^{n+1}$ be an isometric immersion of a Riemannian manifold M^n oriented by a unit vector ν. We call $X = S_c$ grad r the position vector of the immersion with respect to p_o, where $S_c(r) = r$, $\frac{\sin(r\sqrt{c})}{\sqrt{c}}$ or $\frac{\sinh(r\sqrt{-c})}{\sqrt{-c}}$, according $c = 0$, $c > 0$ or $c < 0$. The function $p = -\langle X, \nu\rangle$ will be called the support function of the immersion. We denote $\theta_c = \frac{d}{dr} S_c(r)$.

Theorem 1. ([1]) *Let $x: M^n \to Q_c^{n+1}$ be an isometric immersion of a compact oriented Riemannian manifold M^n with mean curvature H and support function p. Then*

$$Hp - \theta_c$$

does not change sign if and only if $x(M^n)$ is a geodesic sphere.

A proof of this theorem is obtained from the following

Lemma. *In the conditions of Theorem 1, if Δ is the Laplacian of M^n, then*

$$\frac{1}{2}\,\Delta\langle X,X\rangle = -c\,S_c^2\,|\,(\text{ grad } r)^T\,|^2 - n\theta_c(Hp-\theta_c).$$

Theorem 2. *Let $x\colon M^n \to S^{n+1}(c)$ be an isometric immersion of a compact oriented Riemannian manifold M^n into the $(n+1)$-sphere of radius $\frac{1}{\sqrt{c}}$, with unit normal vector ν, mean curvature $H>0$ and support function p. If*

$$H = p,$$

then $x(M^n)$ is a geodesic sphere.

This theorem has been proved by G. Huisken ([2]) when the ambient space is the Euclidean space $\mathbf{R}^{n+1}$.

References

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