Matemática Contemporânea, Vol. 7, 181–183
http://doi.org/10.21711/231766361994/rmc78

γ-HYPERELLIPTICITY AND WEIGHTS OF WEIERSTRASS POINTS

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In this note, M is a compact Riemann surface (or a projective, irreducible non-singular algebraic curve defined over an algebraically closed field of characteristic zero) of genus g and for a point $P \in M$, $m_1, \ldots, m_g$ are the first g positive terms of non-gaps of the Weierstrass semi-group $H(P)$ at P and $w(P)$ is the weight of $H(P)$.

The surface M is called **γ-hyperelliptic** if it is a double covering of a compact Riemann surface of genus γ. If $\gamma = 0$, M is simply called hyperelliptic.

The theorems we present here are part of our Ph.D. dissertation and these are r elated to the following result about hyperelliptics:

If $g \geq 2$, the following are equivalent:
(.) M is hyperelliptic,
(H_1) $\exists P \in M$ such that $m_1 = 2$,
(H_2) $\exists P \in M$ such that $w(P) = \binom{g}{2}$.
With respect to (H_1), we have:

Theorem 1. If $g \geq 6\gamma + 4$, the following are equivalent:

(i) M is γ-hyperelliptic,
(ii) $\exists P \in M$ such that $H(P)$ is a γ-hyperelliptic semigroup.
By a **γ-hyperelliptic semi-group** H we mean a numerical sub-semigroup of $(N, +)$ such that:

. The first γ positive terms $m_1, \ldots, m_\gamma$ are even,

. $m_\gamma = 4\gamma$,

. $4\gamma + 2 \in H$.

Theorem 1 follows from a Castelnuevo's bound and from arithmetical properties of γ-hyperelliptic semigroups. Also, the implication (ii) $\rightarrow$ (i) of this theorem, is a stronger version of a result of T. Kato [3].

With respect to (H_2), we have:

Theorem 2. There exists a polynomial function $F(\gamma)$ such that if $g \geq F(\gamma)$ and $P \in M$, the following are equivalent:

(i) $H(P)$ is t-hyperelliptic for some t between 1 and γ,

(ii)

$$\sum_{i=1}^{g} m_i \leq g^2 + (2\gamma+1)g - \gamma(2\gamma+1)$$

Then, from

$$w(P) = \frac{3g^2+g}{2} - \sum_{i=1}^{g} m_i$$

and from the previous theorems, we have an analogue of (H2):

Theorem 3. There exists a polynomial function $F(\gamma)$ such that if $g \geq F(\gamma)$, the following are equivalent:

(i) M is γ-hyperelliptic,

(ii) $\exists P \in M$ such that $\binom{g-2\gamma}{2} \leq w(P) < \binom{g-2\gamma+2}{2}$.

Part (ii) $\rightarrow$ (i) of theorem 3 was obtained by Kato [2] for the cases $\gamma = 1$ and for $w(P) = \binom{g-2}{2} + 2$. The cases $\gamma = 1, 2$ of the theorem, were proved by Garcia [1].

References

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[3] Kato, T., *On criteria of γ-hyperellipticity*, Kodai Math. J. 2, 275 - 285 (1979).

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