CONFORMAL HYPERSURFACES WITH THE SAME GAUSS MAP

M. Dajczer ⓘ   E. Vergasta

Elie Cartan, in one of his earlier papers in differential geometry, classified all Euclidean hypersurfaces of dimension at least five, which admit conformal deformations other than the trivial ones obtained through compositions with conformal diffeomorphisms of the ambient space. When all of its possible conformal deformations are trivial an hypersurface is called *conformally rigid*. First Cartan proved that an hypersurface is conformally rigid if at no point there exists a principal curvature of multiplicity at least $n-2$ (see **Ca**, **dC-D**, or **Da**). Using this, he concluded that the conformally deformable hypersurfaces are either conformally flat or some special types of 2-parameter envelopes of spheres or planes. Moreover, the set of conformal deformations is either a one parameter family or there is just one other deformation.

In this announcement we classify all Euclidean hypersurfaces which admit nontrivial conformal deformations preserving the Gauss map. In the isometric case, a complete classification for any codimension has been obtained by Dajczer and Gromoll **D-G**, while the conformal case, but only for surfaces, is due to Vergasta **Ve₂**.

Theorem. *Let $f, g : M^n \to R^{n+1}, n \geq 3$, be conformal immersions of an n-dimensional connected Riemannian manifold with the same Gauss map. Assume that on no open subset they differ by a conformal diffeomorphism of R^{n+1}. Then $f(M^n)$ is part of one of the following examples while $g(M^n)$ is of the same type:*

(i) *A minimal real Kaehler hypersurface.*

(ii) *A rotation hypersurface over a plane curve.*

(iii) *A rotation hypersurface over a minimal surface in R^3.*

Minimal real Kaehler hypersurfaces have been completely classified by Dajczer and Gromoll in **D-G**. The assumption that f and g do not differ locally by a conformal deformation of the ambient space has been introduced in order to produce a global result. Without that assumption, the only other possibility is to have an open subset where, up to homotety and rigid motion, either f coincides with g or f is part of a cone and g is obtained by an inversion with respect to the vertex. The classical Liouville's theorem shows up as a special case when the cone is just part of an affine hyperplane.

For hypersurfaces of type (i) or (iii), the set of nontrivial conformal deformations is a 1-parameter family. In fact, for the hypersurfaces of type (i) what we get is the isometric *associated family* of minimal immersions defined in **D -G**. In case (iii) associated to the family of non-isometric deformations, what we obtain is a 1-parameter family of minimal surfaces such that the quocient of the distances to a fixed axis of any two of them is the conformal factor of the correspondent hypersurfaces.

For each hypersurfaces of type (ii) there exists only one conformal deformation preserving the Gauss map. Finally, only one hypersurface belongs simultaneously to two different classes, namely, the one obtained by rotating a catenary, which also belongs to class (iii). Its deformation as an element of class (ii) is the round sphere.

Some partial results related to the present work has been announced in **Ve₁**. Proofs will appear elsewhere.

References

[Ca] Cartan,E., *La déformation des hipersurfaces dans l'espace conforme réel à n ≥ 5 dimensions*, Bull. Soc. Math. France 45 (1917), 57-121.

[dC-D] do Carmo, M. and Dajczer, M., *Conformal rigidity*, Amer. J. of Math.

109 (1983), 963-985.

[Da] Dajczer et al., "Submanifolds and Isometric Immersions", Math. Lec. Series 13, Publish or Perish Inc., Houston, 1990.

[D-G] Dajczer, M. and Gromoll, D., *Real Kaehler submanifolds and uniqueness of the Gauss map*, J. Diff. Geom. 22 (1985), 13-28.

[Ve₁] Vergasta, E., *Conformal immersions with the same Gauss map*, An. Acad. Bras. Cienc. 59 (1987), 145-147.

[Ve₂] Vergasta, E., *Conformal deformations preserving the Gauss map*, Pac. J. of Math. 156 (1992), 359-369.

M. Dajczer

IMPA

Estrada Dona Castorina, 110

22460-320 - Rio de Janeiro - Brazil

E-mail: marcos@impa.br

E. Vergasta

Departamento de Matemática,

Universidade Federal da Bahia

40170-110 - Bahia - Brazil

E-mail: demat04@brufu.bitnet