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THE INTERMEDIATE LONG-WAVE EQUATION IN WEIGHTED SOBOLEV SPACE

Mílton Procópio de Borba

Abstract

We show that the Cauchy problem for the (perturbed) ILW, $u_t + 2uu_x + \mathcal{L}u_x + \lambda u/g = 0$ is globally (on any real interval where g is integrable) well-posed in the topology of $Y_{s,\gamma} = H^s(\mathbf{R}) \cap L^2(\mathbf{R}, w_\gamma^2 dx)$, where $w_\gamma(x) = (1+x^2)^{\gamma/2}$, for any $s > 3/2$ and $0 \le \gamma \le s$, $(\mathcal{L}f)^\wedge = \mathcal{S}\hat{f}$, $\mathcal{S}(\xi) = 1/\alpha - \xi\coth(\alpha\xi) = -\alpha\xi^2/3 + \mathcal{O}(\alpha^3)$ as $\alpha \to 0$ and $\mathcal{S}(\xi) \to |\xi|$ as $\alpha \to \infty$. The solution u_α converges to the solution of the (perturbed) BO equation (as $\alpha \to \infty$) in H^s and $Y_{s,\gamma}$, for $1/2 < \gamma \le 2$, while $\tilde{u}_\alpha$ approaches the solution of the (perturbed) KdV equation (as $\alpha \to 0$) in $Y_{s,\gamma}$ for $0 \le \gamma \le s/2$, where $\tilde{u}(t) = 3u(3t/\alpha)/\alpha$. For $2 < \gamma \in \mathbf{N}$, the limit $\alpha \to \infty$ holds if $u_\alpha(t,x)$ is such that $\partial_\xi^j \hat{u}_\alpha(0,t) = 0$ for every $j \in \{0, 1, \ldots, \gamma - 3\}$. For $j \ge 1$, the limit is zero.

Notation

$w_\gamma(x) = (1+x^2)^{\gamma/2}, \quad \gamma \in \mathbf{R}, \quad \gamma \ge 0.$

$J^s = (1-\Delta)^{s/2}, \quad s \in \mathbf{R}.$

$\hat{f} = \ell \cdot i \cdot m \cdot \frac{1}{\sqrt{2\pi}} \int_{\mathbf{R}} f(x) e^{-i\xi \cdot x} dx =$ **Fourier Transform**.

$L^p = L^p(\mathbf{R}),\ |f|_{L^p}^p = \int_{\mathbf{R}} |f|^p\, dx, \quad 1 \le p < \infty.$

$H^s = H^s(\mathbf{R}) =$ **Sobolev Space**, $|f|_{H^s} = |w_s \hat{f}|_{L^2} = |J^s f|_{L^2}$.

$P_\gamma = L^2(\mathbf{R}, w_\gamma^2 dx) =$ **Weighted L^2 Space**, $|f|_{P_\gamma} = |w_\gamma f|_{L^2}$.

$Y_{s,\gamma} = H^s \cap P_\gamma =$ **Weighted Sobolev Space**, $|f|_{Y_{s,\gamma}}^2 = |f|_{H^s}^2 + |f|_{P_\gamma}^2$.

Simplification

$|\cdot|_{s,\gamma} = |\cdot|_{Y_{s,\gamma}}$

$|\cdot|_s = |\cdot|_{s,0} = |\cdot|_{H^s}$

$|\cdot| = |\cdot|_0 = |\cdot|_{0,0} = |\cdot|_{L^2} = |\cdot|_{P_0}$

$(\cdot,\cdot)_z = z$-**inner product**.

1. Introduction

In this work, we consider the Cauchy problem

$$\begin{cases} u \in C(I, Y_{s,\gamma}) \\ u_t + 2uu_x + Lu + \lambda u/g = 0, \quad t \in I \setminus \{t_n\} \\ u(t_0) = \phi \end{cases} \tag{1.1}$$

where $\{t_n\} = \{t_0, t_1, \ldots\} \subset I$ is a discrect sequence with $\{t_n\} = \overline{\{t_n\}}$, $I = I_g$ is a real interval such that $\lambda/g \in L^1(I)$, $g\colon I \setminus \{t_n\} \to \mathbf{R} \setminus \{0\}$ is continuous, x, t, λ and $u = u(x,t)$ are reals, and L is one of the following operators

$$\begin{array}{rc} (g - ILW) & Lu = u_x/\alpha + T_\alpha u_{xx}, \quad \alpha > 0 \\ (g - BO) & Lu = \sigma u_{xx} \\ (g - KdV) & Lu = u_{xxx} \\ (g - S) & Lu = (J-1)u_x \\ (g \sim ILW) & Lu = 3(u_x/\alpha + T_\alpha u_{xx})/\alpha, \end{array}$$

with

$$T_\alpha f(x) = \frac{1}{2\alpha} p.v. \int_{\mathbf{R}} \coth\left[\frac{\pi(y-x)}{2\alpha}\right] f(y)dy, \tag{1.2}$$

$$\sigma f(x) = \frac{1}{\pi} p.v. \int_{\mathbf{R}} \frac{f(y)}{y-x} dy. \tag{1.3}$$

According to several authors (see [J], [KKD], [JE], [CL], [B], [O], [KdV], [S]), $(g - ILW)$ describes long internal gravity waves in stratified fluids of finite depth (α), $(g - BO)$ represents the deep water limit $(\alpha \to \infty)$, $(g - KdV)$ represents the shallow water limit $(\alpha \to 0)$ and $(g - S)$ governs the continental-shelf waves.

We can resume this work in three main results:

Result 1.1. *The problem (1.1) is globally well-posed.*

Result 1.2. $(g - ILW) \to (g - BO)$, *as* $\alpha \to \infty$.

Result 1.3. $(g \sim ILW) \to (g - KdV)$, *as* $\alpha \to 0$.

2. Well-posedness

In this section, we present (in an exact sense) the Result 1.1 and give a sketch of its proof.

Theorem 2.1 *(existence-uniqueness) If $s > 3/2$, $0 \le \gamma \le s$ and $\phi \in Y_{s,\gamma}$, then there exists a unique solution u for $(g - ILW)$. Moreover,*

$$\partial_x^k u \in C(I, P_r) \quad \text{if } s \ge k \in \mathbf{N} \text{ and } 0 \le r \le \gamma(1 - k/[s]). \tag{2.1}$$

Sketch of proof: We use the parabolic regularization

$$\begin{cases} u_t + 2uu_x + Lu + \lambda u/g = \mu u_{xx}, & \mu \neq 0 \\ u(0) = \phi, \end{cases} \tag{2.2}$$

and write (2.2) in the integral form

$$u(t) = G(0,t)E(t)\phi - \int_0^t G(\tau,t)E(t-\tau)[u(\tau)^2]_x d\tau,$$

where

$$E(t) = e^{-t(L-\mu\partial_x^2)}, \quad G(\tau,t) = \exp\left[-\int_\tau^t \frac{\lambda}{g(s)} ds\right].$$

The properties of $t \mapsto E(t)$ and the Fixed Point Theorem show that there exists a small interval $I(= I_\mu)$ dependent on μ such that the theorem holds for $\mu \neq 0$. By using a Kato's inequality (see [K], Lemma A5), $u = u_\mu$ can be extended to some I_0 interval (independent on μ).

A limit argument ($\mu \to 0$) (see [I3], sketch of the proof for Theorem 2.1) implies that the theorem holds in the interval I_0. In order to extend u to I, once again we use parabolic regularization, the Hamiltonian structure

$$u_t = \partial_x \psi'(u) + \mu u_{xx} - \lambda u/g \tag{2.3}$$

and the conserved quatities ψ and φ such that

$$(\varphi'(u), \partial_x \psi'(u)) = 0, \tag{2.4}$$

where φ' is the directional derivative given by

$$(\varphi'(u), f) = \frac{d}{dz}\varphi(u + zf)|_{z=0}.$$

So, by (2.3) and (2.4), we have

$$\frac{d}{dt}\varphi(u(t)) = (\varphi'(u), u_t) = \mu(\varphi'(u), u_{xx}) + \frac{\lambda}{g}(\varphi'(u), u).$$

By integration over $[0,t]$, we obtain

$$\varphi(u(t)) = \varphi(\phi) + \mu\int_0^t (\varphi'(u), u_{xx})d\tau + \lambda\int_0^t \frac{(\varphi'(u), u)}{g(\tau)}d\tau.$$

According to [ABFS], $\varphi(u) = \mid u \mid_s + R$. Moreover, R, $(\varphi'(u), u_{xx})$ and $(\varphi'(u), u)$ can be estimated in terms of $\mid u \mid_s$. Therefore, the Gronwall's inequality implies that

$$\mid u_\mu \mid_s \leq C_s(t, \mid \phi \mid_s, \mid \lambda/g \mid_{L^1}), \quad \forall t \in I, \text{ independent over } \mid \mu \mid \leq 1. \tag{2.5}$$

In order to estimate $\mid u_\mu \mid_{0,\gamma}$, we use linear interpolation and obtain the following estimative for the commutator

$$\mid [w_\gamma, L]f \mid \leq C(\gamma) \mid f \mid_{s,\gamma}, \quad \forall f \in Y_{s,\gamma} \tag{2.6}$$

Now we consider $u = u_\mu$ and obtain

$$\frac{d}{dt} \mid u(t) \mid_{0,\gamma}^2 = 2\mu(u, u_{xx})_{0,\gamma} - 4(u, uu_x)_{0,\gamma} - \frac{2\lambda}{g}(u,u)_{0,\gamma} - 2(u, Lu)_{0,\gamma}. \tag{2.7}$$

Since $\widehat{Lf} = -ih\hat{f}$, with h odd and u is real, we have

$$\mid (u, Lu)_{0,\gamma} \mid = \mid (w_\gamma u, Lw_\gamma u + [w_\gamma, L]u) \mid \leq \mid u \mid_{0,\gamma} C(\gamma) \mid u \mid_{s,\gamma}.$$

The other three terms of (2.7) are easier to estimate. It follows from (2.7), by integration over $[0,t]$ and Gronwall's inequality, that

$$\mid u_\mu \mid_{0,\gamma} \leq C_\gamma(t, \mid \phi \mid_{s,\gamma}, \mid \lambda/g \mid_{L^1}), \quad \forall t \in I, \text{ independent over } \mid \mu \mid \leq 1. \tag{2.8}$$

Once again, a limit argument ($\mu \to 0$), with (2.5) and (2.8), can show that

$$\mid u(t) \mid_{s,\gamma} \leq C_{s,\gamma}(t, \mid \phi \mid_{s,\gamma}, \mid \lambda/g \mid_{L^1}), \quad \forall t \in I.$$

The last statement is a consequence of [K], Lemma A7.

Theorem 2.2 *(continuous dependence) Let $s > 3/2$, $0 \leq \gamma \leq s$. Given a compact interval $I_c \subset$, if $\phi_n \to \phi$ in $Y_{s,\gamma}$, $\frac{\lambda_n}{g_n} \to \frac{\lambda}{g}$ in $L^1(I_c)$ and $\alpha_n \to \alpha$, then there is a integer N_0 such that $\forall n \geq N_0$, $t \in I_c$, there exists $u_n(t)$, the correspondent solution for $g_n - ILW$,*

$$\sup_{I_c} \mid u_n(t) - u(t) \mid_{s,\gamma} \to 0 \text{ as } n \to \infty,$$

and (2.1) implies

$$\sup_{I_c} \mid \partial_x^k [u_x(t) - u(t)] \mid_{0,r} \to 0 \text{ as } n \to \infty.$$

Sketch of proof: We consider the regularization of the initial data ϕ_ε given by $\hat{\phi}_\varepsilon = \varphi_\varepsilon \hat{\phi}$, where $\varphi_\varepsilon(\xi) = \varphi(\varepsilon\xi)$ and $\varphi: \mathbf{R} \to [0,1]$ is such that $\varphi \in \mathcal{S}(\mathbf{R})$ and $\varphi^j(0) = 0$, $\forall j \geq 1$.

Now, let u^ε be the solution of

$$\begin{cases} u_t + 2uu_x + Lu + \lambda u/g = 0 \\ u(0) = \phi_\varepsilon \end{cases} \tag{2.9}$$

As in [ABFS], $u^\varepsilon \to u$ and $u_n^\varepsilon \to u_n$ (as $\varepsilon \to 0$) uniformly on n (see Proposition 5.3.6), in the $L^\infty(I_c, H^s)$ topology. Since u_n^ε also approaches u^ε (as $n \to \infty$) in $L^\infty(I_c, H^s)$, we complete the argument by

$$\mid u_n - u \mid_s \leq \mid u_n - u_n^\varepsilon \mid_s + \mid u_n^\varepsilon - u^\varepsilon \mid_s + \mid u^\varepsilon - u \mid_s .$$

To obtain a similar result for P_γ-norm, we consider the following regularization

$$\phi^j = \varphi_{1/j}[\varphi_{1/j}\hat{\phi}]^\vee \to \phi \text{ in } Y_{s,\gamma}, \text{ as } j \to \infty,$$

and use continuous dependence in $L^\infty(I_c, H^s)$.

3. The α-limits

By using arguments similar to those described in the preceding sketch of proof, we show the following results:

Theorem 3.1 *(BO-limit) Let $\gamma = 0$, or $1/2 < \gamma \leq 2$, $\phi \in Y_{s,\gamma}$, and $I_c \subset I$ be a compact interval. Then there exists $u_\infty(t)$, solution for $g - BO$, $\forall t \in I_c$, and*

$$\sup_{I_c} \mid u_\alpha(t) - u_\infty(t) \mid_{s,\gamma} \to 0 \text{ as } \alpha \to \infty,$$

where u_α is the solution for $g - ILW$, and

$$\sup_{I_c} \mid \partial_x^k [u_\alpha(t) - u_\infty(t)] \mid_{0,r} \to 0 \text{ as } \alpha \to \infty$$

if (2.1) holds.

Remark 3.2 According to [I1], we can also obtain that for $\gamma \in \{3, 4, 5, \ldots\}$, Theorem 3.1 holds if and only if $\partial_\xi^j \hat{\phi}(0) = 0$, $\forall j \in \{0, 1, \ldots, \gamma - 3\}$, and $\gamma \geq 4$ implies $u(t) = 0$, $\forall t \in I_c$.

Theorem 3.3 *(KdV-limit) Let $s \geq 2$, $0 \leq \gamma \leq s/2$ and $\phi \in Y_{s,\gamma}$. Consider $I = \mathbf{R}$ or $I = [0, +\infty)$ or $I = (-\infty, 0]$, $\lambda/g \in L^1(I)$ or $\lambda/g \in L^1_{loc}(I)$ and $\lim_{t\to\infty} \frac{\lambda}{tg(t)} < \infty$, and define*

$$\tilde{g}(t) = 3g(3t/\alpha)/\alpha, \quad \tilde{u}(x,t) = 3u_\alpha(x, 3t/\alpha)/\alpha,$$

where u_α is the solution for $g - ILW$. Then given a compact interval $I_c \subset I$, there exists $\tilde{u}_\alpha(t)$, solution for $\tilde{g} \sim ILW$, $\forall t \in I_c$, and

$$\sup_{I_c} \mid \tilde{u}_\alpha(t) - u_0(t) \mid_{s,\gamma} \to 0 \text{ as } \alpha \to 0,$$

where u_0 is the solution for $\tilde{g} - KdV$. Moreover, (2.1) implies

$$\sup_{I_c} \mid \partial_x^k [\tilde{u}_\alpha(t) - u_0(t)] \mid_{0,r} \to 0 \text{ as } \alpha \to 0.$$

Note that $\widehat{Lf} = -ih_\alpha \hat{f}$, with

$$h_\alpha(\xi) = \xi^2 \coth(\alpha\xi) - \xi/\alpha \to \xi^2 \operatorname{sgn}(\xi) = h_\infty(\xi), \text{ as } \alpha \to \infty$$

while (see figure 1, where $S = h_1$)

$$h_\alpha(\xi) = \frac{\alpha}{3}(\xi^3) + \mathcal{O}(\alpha^3) \text{ as } \alpha \to 0.$$

This is sufficient to study the H^s-limit. In order to consider P_γ-limit, observe that

$$h'_\alpha(\xi) \to 2(\xi) = h'_\alpha(\xi), \quad h''_\alpha(\xi) \to 2\mathrm{sgn}(\xi) = h''_\infty(\xi),$$

$$h'''_\alpha(\xi) \to 4\delta_0 = h'''_\infty(\xi), \text{ as } \alpha \to \infty,$$

and the derivatives of h_α approach the derivatives of $\frac{\alpha}{3}(\xi^3)$, close to the origin (see figure 2, 3 and 4).

4. Some previous results

4.1 with $\lambda = 0$, in H^s

The first results about KdV, BO and S were obtained in (KdV – 1895) D.J. Korteweg and G. de Vries – [KdV]; (BO – 1967) T.B. Benjamin – [B]; (1977) H. Ono – [O]; (S – 1972) R. Smith – [S].

The ILW equation has arised in 1977, when R.I. Joseph [J] analysed its solitons, i.e., solutions of the form $u(x,t) = \varphi(x - \tilde{c}t)$, where $\varphi(y) = C/[\cosh^2(ay) + c^2\sinh^2(ay)/16a^2]$, $a\mathrm{tg}(a\alpha) = c/4$, and $\tilde{c} = 1/\alpha - 4a^2/c + c/4 + 1$.

This equation was also considered in 1978 by T. Kubota, K.R.S.Ko, D. Dobbs [KKD], R.I. Joseph and R. Egri [JE], and in 1979, by H.H. Chen and Y.C. Lee [CL].

The Hamiltonian Structure and Conservation Laws for the ILW equation were studied in 1979–83 by J. Satsuma, M.J. Ablowitz, Y. Kodama [SAK1] [SAK2], D.R. Lebedev and A.O. Radul [LR].

The first limits ($ILW \to BO$, $ILW \to KdV$) were considered by J.P. Albert, J.L. Bona and D.B. Henry [ABH], in 1986, in the study on stability of the solitons for the ILW equation, with α close to zero and α very large.

Recently (1989), L. Abdelonhab, J. Bona, M. Felland and J. Saut [ABFS] have studied the limits $\alpha \to 0$ and $\alpha \to \infty$ as in Theorem 3.1 and 3.3 in H^s, using parabolic regularization of the form $\mu + \mu\partial_x^4$, with regularization ϕ^ϵ of the initial data ϕ.

4.2 with $\lambda \neq 0$, in H^s

According to S. Maxon [M] (1978), F. Calogero and A. Degasperis [CD] (1982), the coefficient $\lambda/g(t)$ is related to the change in the depth of botton layer of the stratified fluid. In the case of KdV, with $\lambda/g(t) = 1/t$ and $\lambda/g(t) = 1/2t$, we obtain the spherical KdV and cylindrical KdV respectively.

Note that, with

$$\frac{\lambda}{g(t)} = -\frac{f'(t)}{f(t)}, \quad u(t) = f(t)v(t),$$

problem (1.1) becomes

$$\begin{cases} v \in C(I, Y_{s,\gamma}) \\ v_t + 2f(t)vv_x + Lv = 0 \\ v(t_0) = f(t_0)\phi, \end{cases}$$

which, in the KdV and BO cases, was studied (in H^s) by W.V.L. Nunes [N], in 1991.

4.3 with $\lambda = 0$, in $Y_{s,\gamma}$

In 1983, T. Kato [K] studied the KdV equation in $Y_{2n,n}$, $n \in \mathbf{N}$. The BO equation (except continuous dependence) in $Y_{n,n}$, $n \in \mathbf{N}$, has been considered by R.J. Iório Jr., in 1986 [I1]. In 1989, R.J. Iório Jr. [I2] has considered the BO equation in $Y_{2,\gamma}$, $0 \leq \gamma \leq 2$, using Kato's theory for linear equations of "hyperbolic" type, to obtain continuous dependence. By using nonlinear interpolation theorem of Tartar, Bona and Scott, in 1990, R.J. Iório Jr. [I3] obtained well-posedness of the Cauchy problem for S equation in $Y_{s,\gamma}$, $s > 3/2$, $0 \leq \gamma \leq s$. The BO and KdV equations have been also considered. Finally, in 1990, L. Abdelonhab [A] analysed the Cauchy problem

$$\begin{cases} u_t + 2uu_x + Lu + f(x,t) = \varepsilon J^k u \\ u(0) = \phi_\varepsilon \in H^r_\gamma \cap H^s, \end{cases}$$

where $H^r_\gamma = (w_\gamma J^r)^{-1} L^2(\mathbf{R})$, $k \geq 2$ for the ILW and BO cases, and $k \geq 3$ for the KdV case. He has also studied the limits $ILW \to BO$ and $ILW \to KdV$ in $L^\infty(I_c, H^r_\gamma \cap H^r)$.

fig. 1 fig. 2

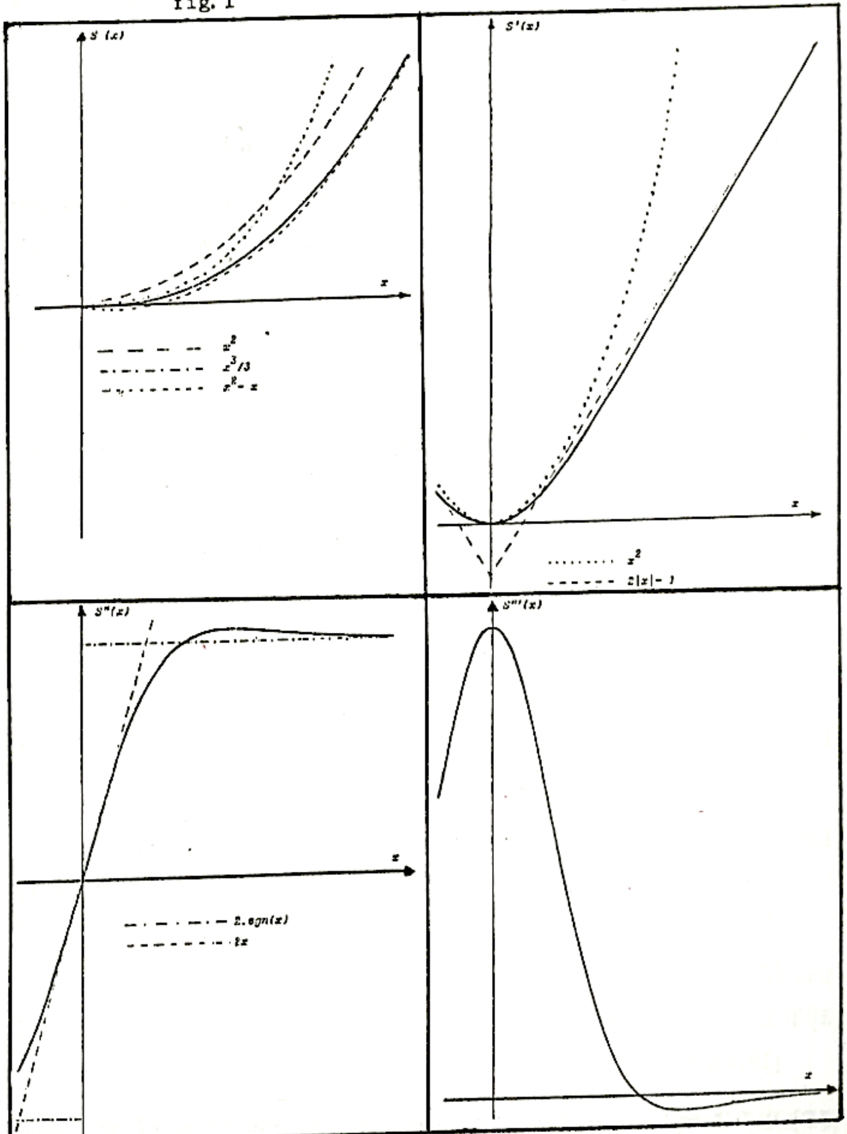

fig. 3 fig. 4

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Faculdade de Engenharia de Joinville
Universidade do Estado de Santa Catarina
Campus Universitário – Bom Retiro
89.200 – Joinville – SC – Brazil