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A PARAMETER DETERMINATION PROBLEM FOR A LINEARIZED MODEL OF WELL-RESERVOIR COUPLING

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Abstract

We consider a linearized model for a well-oil reservoir coupling problem. We suppose that the permeability k is a function of the space variable. Under appropriate regularity assumptions on k we show that a stable solution exists. This result extends those obtained in [5] for constant k. We also show that, k being constant, the knowledge of the pressure p at the well over a time interval determines k we show existence.

Resumo

Consideramos um modelo linearizado para um problema de acoplamento poço-reservatório de petróleo. Mostramos, sob hipóteses adequadas, a existência de solução no caso em que a permeabilidade k é variável no espaço. O comportamento assintótico da solução é analisado, mostrando que o modelo é estável. Esses resultados estendem aqueles obtidos anteriormente para k constante [5]. Retornando à hipótese k constante, mostramos ainda que, dado $T > 0$, o conhecimento da pressão p ao longo do poço e durante o intervalo de tempo $(0, T)$ determina k univocamente.

1. Introduction

Oil reservoir engineering devotes a great attention to the well-testing problem where, by controlling oil injection and/or production and by measuring the well pressure evolution, one tries to determine the relevant fluid and rock properties. Among them, rock permeability plays a crucial role. Simplified models of diffusion type scalar equations have been considered to simulate this coupled well-reservoir flow. Since the early seventies, however, it has been recognized that this simplifications would result in misleading interpretations of the reservoir response. Some ad-hoc factors, in particular a storage coefficent [1], have been then incorporated to the models. Most recently yet, with the appearance of new technological trends, as the utilisation of non-vertical wells, more involved system of equations, which dispense the use of storage parameters, have been proposed [4], [2].

In this paper we discuss some theoretical aspects related to those new models. Stability and convergence results for finite-differences numerical approximations of this problem is discussed in [6]. In Section 2 we describe some results obtained in [5] for a constant coefficient linearization of the system proposed in [4] and show that they can be extended when k, the rock permeability, is not constant. In Section 3 we introduce a permeability determination problem. We take k to be constant and show, under some reasonable hypotheses, that the knowledge of the well pressure over any time interval determines k.

2. The coupled model.

Let $\Omega = (0,1)\times(0,1)$, $\Gamma_0 = \{0\}\times[0,1]$ and $\Gamma = \partial\Omega\setminus\Gamma_0$. Given $p_0, v_0 : (0,1)\to\mathbf{R}$, $P_0:\Omega\to\mathbf{R}$ and $\alpha:(0,T)\to\mathbf{R}$, we consider the problem of finding $p, v:(0,T)\times(0,1)\to\mathbf{R}$ and $P:(0,T)\times\Omega\to\mathbf{R}$ such that:

$$\begin{cases} p_t + v_y + k\frac{\partial P}{\partial n}_{|\Gamma_0} = \alpha(t) \quad \text{in} \quad (0,T)\times(0,1), \\ v_t + p_y = -(1-y)\alpha'(t) \quad \text{in} \quad (0,T)\times(0,1), \\ p(0,y) = p_0(y) \quad v(0,y) = v_0(y) \quad \text{for} \quad y\in(0,1), \\ v(t,0) = v(t,1) = 0 \quad \text{for} \quad t\in(0,T), \end{cases} \tag{2.1}$$

and

$$\begin{cases} P_t - \nabla\cdot(k\nabla P) = 0 \quad \text{in} \quad (0,T)\times\Omega, \\ P(t,x,y) = p(t,y) \quad \text{for} \quad (t,x,y)\in(0,T)\times\Gamma_0, \\ \frac{\partial P}{\partial n}(t,x,y) = 0 \quad \text{for} \quad (t,x,y)\in(0,T)\times\Gamma, \\ P(0,x,y) = P_0(x,y) \quad \text{for} \quad (x,y)\in\Omega, \end{cases} \tag{2.2}$$

Equations (2.1)–(2.2) are obtained by linearization around an equilibrium state solution of the system proposed in [4] and by considering a standard change of variables leading to homogeneous boundary conditions (see [5]). Here, Ω is the reservoir region, Γ_0 the wellbore, k the permeability, p the well pressure, v the well flow velocity and P the reservoir pressure. Considering the fluid state equation relating density and pressure as $\rho(p) = p$, the first equation in (2.1) expresses mass conservation. Then, $k\frac{\partial P}{\partial n}_{|\Gamma_0}$ is the well-reservoir mass flow exchange given by Darcy's law [7], where gravity effects have been neglected. Frequently (see [3] and [7]) the diffusion equation has been considered as a simplified model for the reservoir flow. The second equation in (2.2) is a continuity condition for the pressure at the wellbore. No-flow is assumed at the reservoir boundary and oil is produced/injected at a given mass flow rate $\alpha(t)$.

We consider the following functional framework. Set

$$H = L^2(0,1) \times L^2(0,1) \times L^2(\Omega),$$

$$D(A) = \left\{ \begin{array}{l} (p, v, P) \in H^1(0,1) \times H_0^1(0,1) \times H^1(\Omega); \nabla\cdot \\ (k\nabla P) \in L^2(\Omega), \frac{\partial P}{\partial n}_{|\Gamma} = 0, P_{|\Gamma_0} = p \end{array} \right\}, \tag{2.3}$$

and

$$A(p, v, P) = (v_y + k\frac{\partial P}{\partial n}_{|\Gamma_0}, p_y, -\nabla \cdot (k\nabla P)), \tag{2.4}$$

for $(p, v, P) \in D(A)$. The function $F : (0,T) \to H$ is defined by

$$F(t) = (\alpha(t), -(1-y)\alpha'(t), 0). \tag{2.5}$$

Then we write (2.1)–(2.2) as

$$\left\{ \begin{array}{l} U'(t) + AU(t) = F(t), \\ U(0) = U_0. \end{array} \right. \tag{2.6}$$

In order to discuss the non-constant permeability case, we take $k \in W^{1,\infty}(\Omega)$ with $k(x) \geq k_* > 0$ for $x \in \Omega$. Then we have the following results.

Theorem 2.1. *Assume $k \in W^{1,\infty}(\Omega)$ with $k(x) \geq k_* > 0$ for all $x \in \Omega$, and let A be defined by (2.3)–(2.4), $T > 0$ and $\alpha \in W^{1,1}(0,T)$.*

(i) *Given $U_0 = (p_0, v_0, P_0) \in H$, there exists a unique solution $U = (p, v, P) \in C([0,T], H)$ of (2.6). In addition, $P \in L^2((0,T), H^1(\Omega))$.*

(ii) *If* $\alpha \in W^{2,1}(0,T)$ *and* $U_0 \in D(A)$, *then* $U \in C([0,T], D(A)) \cap W^{1,1}((0,T), H)$ *and* U *solves the equation (2.6) for almost all* $t \in (0,T)$. *If furthermore* $\alpha \in C^2([0,T])$, *then* $U \in C^1([0,T], H)$.

Theorem 2.2. *Assume that* $\alpha \in W^{1,1}_{\mathrm{loc}}(0,\infty)$ *and that* $\int_0^t \alpha(s)\,ds$ *is bounded as* $t \to \infty$.

(i) *If* $\sup\limits_{t\geq 0} \int_t^{t+1} (|\alpha(s)| + |\alpha'(s)|)\,ds < \infty$, *then for every* $U_0 \in H$ *the solution* U *of (2.6) is bounded, i.e.* $\sup\limits_{t\geq 0} \|U(t)\|_H < \infty$. *Moreover, if* U_1 *and* U_2 *are two solutions, with* $U_i(0) = U_{i0}$, *then* $U_1(t) - U_2(t)$ *converges to* 0 *in* H *as* $t \to \infty$.

(ii) *If in addition* α *is* τ*-periodic for some* $\tau > 0$, *then there exists a* τ*-periodic solution* U *of (2.6).*

The two theorems above have been proved in [5] for constant k. It is not difficult to verify that the same arguments hold here, with small modifications, except for the following lemma.

Lemma 2.3. *Assume* $k \in W^{1,\infty}(\Omega)$ *with* $k(x) \geq k_* > 0$ *for all* $x \in \Omega$. *Given* $f \in L^2(\Omega)$ *and* $p \in H^1(0,1)$, *there exists a unique solution* $u \in H^1(\Omega)$ *of the equation*

$$\begin{cases} -\nabla \cdot (k\nabla u) = f & \text{in} \quad \Omega, \\ \frac{\partial u}{\partial n} = 0 \quad \text{in} \quad \Gamma, \\ u = p \quad \text{in} \quad \Gamma_0. \end{cases} \tag{2.7}$$

The mapping $(f,p) \mapsto u$ *is continuous* $L^2(\Omega) \times H^1(0,1) \to H^1(\Omega)$. *Furthermore,* $\frac{\partial u}{\partial n}_{|\Gamma_0}$ *is well defined,* $\frac{\partial u}{\partial n}_{|\Gamma_0} \in L^2(0,1)$, *and the mapping* $(f,p) \mapsto \frac{\partial u}{\partial n}_{|\Gamma_0}$ *is continuous* $L^2(\Omega) \times H^1(0,1) \to L^2(0,1)$. *Moreover,*

$$\int_\Omega k\nabla u \cdot \nabla v = \int_\Omega fv + \int_{\Gamma_0} kv\frac{\partial u}{\partial n}, \tag{2.8}$$

for all $v \in H^1(\Omega)$.

Proof. We proceed in two steps.

Step 1. A stability result. Given $g \in L^2(\Omega)$, $p \in H^1(\Gamma_0)$, suppose $u \in H^1(\Omega)$ satisfies

$$\begin{cases} -\triangle u = g & \text{in} \quad \Omega, \\ \frac{\partial u}{\partial n} = 0 & \text{in} \quad \Gamma, \\ u = p & \text{in} \quad \Gamma_0. \end{cases}$$

Then there exists $C > 0$ such that

$$\|\nabla u\|_{L^2(\Omega)} \leq C(\|g\|_{L^2(\Omega)} + \|p\|_{H^1(\Gamma_0)}). \tag{2.9}$$

In fact, let $v(x,y) = u(x,y) - p(y)$. So $v \in H^1(\Omega)$ and $v_{|\Gamma_0} = 0$. Therefore,

$$\int_\Omega \nabla u \nabla v = \int_\Omega gv,$$

gives

$$\int_\Omega \nabla u \nabla u - \int_\Omega \frac{\partial u}{\partial y}(x,y)p'(y)dxdy = \int_\Omega g(u-p),$$

and so

$$\|\nabla u\|^2_{L^2(\Omega)} \leq \|p\|_{H^1(\Gamma_0)}\|\nabla u\|_{L^2(\Omega)} + \|g\|_{L^2(\Omega)}(\|u\|_{L^2(\Omega)} + \|p\|_{H^1(\Gamma_0)}).$$

Inequality (2.9) follows easily.

Step 2. Conclusion. Let $g = \nabla k \cdot \nabla u \in L^2(\Omega)$. We can write

$$-\triangle u = \frac{1}{k}(f+g).$$

Therefore, using Lemma 3.1 of [5], which is constant coefficient analogous of Lemma 2.3, we obtain that $\dfrac{\partial u}{\partial n}_{|\Gamma_0}$ is a well defined function of $L^2(\Gamma_0)$ verifying

$$\left\|\frac{\partial u}{\partial n}\right\|_{L^2(\Gamma_0)} \leq C_1(\|f\|_{L^2(\Omega)} + \|g\|_{L^2(\Omega)} + \|p\|_{H^1(\Gamma_0)}).$$

From the definition of g and (2.9) it follows that

$$\left\|\frac{\partial u}{\partial n}\right\|_{L^2(\Gamma_0)} \leq C_2(\|f\|_{L^2(\Omega)} + \|p\|_{H^1(\Gamma_0)}).$$

Moreover, from Lemma 3.1 of [5] we have

$$\int_\Omega \nabla u \cdot \nabla w = \int_\Omega \frac{1}{k}(f + \nabla k \nabla u)w + \int_{\Gamma_0} w\frac{\partial u}{\partial n},$$

for all $w \in H^1(\Omega)$. If $v \in H^1(\Omega)$, we take $w = kv \in H^1(\Omega)$ in the above inequality to obtain (2.8).

$\square$

Remark 2.4. If $U = (p, v, P) \in D(A)$ then

$$(AU, U)_H = \int_{\Gamma_0} \left(v_y + k \frac{\partial P}{\partial n}_{|\Gamma_0} \right) p + \int_{\Gamma_0} p_y v - \int_\Omega P \nabla \cdot (k \nabla P);$$

and so (2.8) gives

$$(AU, U)_H = \int_\Omega k |\nabla P|^2.$$

Therefore, A is a positive-definite (but not strictly) linear operator on H. The fact that A is m-accretive, which still holds here, is on the basis of the proofs of Theorems 2.1 and 2.2.

Remark 2.5. The conclusions of Lemma 2.3 do not hold if we assume only $k \in L^\infty(\Omega)$. Indeed, using polar coordinates, let

$$k(\theta) = \begin{cases} 1 & \text{if } \ 0 \le \theta < \frac{\pi}{4}, \\ \tan^2 \frac{\pi}{8} & \text{if } \ \frac{\pi}{4} \le \theta \le \frac{\pi}{2}; \end{cases}$$

$$g(\theta) = \begin{cases} \cos \frac{\theta}{2} & \text{if } \ 0 \le \theta < \frac{\pi}{4}, \\ \cot \frac{\pi}{8} \sin \left(\frac{\pi}{4} - \frac{\theta}{2} \right) & \text{if } \ \frac{\pi}{4} \le \theta \le \frac{\pi}{2}; \end{cases}$$

and

$$u(r, \theta) = r^{1/2} g(\theta).$$

Then, $u \in H^1(\Omega)$, $u_{|\Gamma_0} = 0$, $\dfrac{\partial u}{\partial n}(r, 0) = r^{-1/2} g'(0) = 0$ and $(kg')' + \frac{1}{4} kg = 0$. Therefore,

$$\nabla \cdot (k \nabla u) = \frac{1}{r} (k r u_r)_r + \frac{1}{r^2} (k u_\theta)_\theta = \frac{1}{r^{3/2}} \left\{ (kg')' + \frac{1}{4} kg \right\} = 0 \in L^2(\Omega),$$

but

$$\frac{\partial u}{\partial n}_{|\Gamma_0} = -\frac{1}{r} \frac{\partial u}{\partial \theta}(r, \frac{\pi}{2}) = \frac{1}{2 r^{1/2}} \cot \frac{\pi}{8} \cos \frac{\pi}{8} \notin L^2(\Gamma_0).$$

Choosing $\varphi \in C^2(\Omega)$ with $\varphi \equiv 1$ on $[0, 1/2] \times [0, 1/2]$, supp$\varphi \subset [0, 3/4] \times [0, 3/4]$, and $\dfrac{\partial \varphi}{\partial y}_{|y=0} = 0$, it follows that $v = \varphi u \in H^1(\Omega)$, $\nabla \cdot (k \nabla v) \in L^2(\Omega)$, $\dfrac{\partial v}{\partial n} = 0$ on Γ, $v = 0$ on Γ_0 and $\dfrac{\partial v}{\partial n}_{|\Gamma_0} \notin L^2(\Gamma_0)$.

3. Permeability determination.

Since porous media flow is essentially driven by Darcy's law, permeability evaluation plays a central role in oil reservoir engineering. In this section we suppose that k is constant, $U_0 = 0$ and $\alpha(0) \neq 0$. Those conditions at $t = 0$ are in fact the most usual ones in well testing. We show that, given any $T > 0$, the knowledge of $p(t)$ for $0 < t < T$ determines k. More precisely, we have the following result.

Theorem 3.1. *Assume k is constant. Let $T > 0$, $U_0 = 0$, $\alpha \in C^2([0,T])$ with $\alpha(0) \neq 0$. For $k > 0$, let $U_k = (p_k, v_k, P_k)$ be the solution of (2.6). Then the mapping $k \mapsto p_k$ is injective $\mathbf{R}^+ \setminus \{0\} \to C^1([0,T]; L^2(\Gamma_0))$.*

Proof. From Theorem 2.1, $U_k \in C([0,T]; D(A)) \cap C^1([0,T]; H)$ and (2.1)–(2.2) is satisfied at $t = 0$. Since $U_0 = 0$, we deduce from the first equation of (2.1) that $\dfrac{\partial p_k}{\partial t}(0, y) \equiv \alpha(0)$. We may assume without loss of generality that $\alpha(0) > 0$, and since $p_k \in C^1([0,T], L^2(0,1))$, it follows that there exists $t_k > 0$ such that

$$\int_{\Gamma_0} \frac{\partial p_k}{\partial t}(t, y)\, dy > 0, \tag{3.1}$$

for all $t \in [0, t_k]$.

For simplicity of notation we write now $U = U_k$. Let $\phi(t)$ be the solution of

$$\begin{cases} -\Delta\phi = 0 & \text{in } \Omega, \\ \frac{\partial \phi}{\partial n} = 0 & \text{in } \Gamma, \\ \phi = p & \text{in } \Gamma_0. \end{cases}$$

Writing $p(t, y) = \sum_n p_n(t) \cos(n\pi y)$, one verifies that

$$\phi(t, x, y) = \sum_n p_n(t) \frac{\cosh(n\pi(1 - x))}{\cosh(n\pi)} \cos(n\pi y). \tag{3.2}$$

Set $Q = P - \phi$. Then, $Q(0) = 0$ and

$$\begin{cases} Q_t - k\Delta Q = -\phi_t & \text{in } \Omega, \\ \frac{\partial Q}{\partial n} = 0 & \text{in } \Gamma, \\ Q = 0 & \text{in } \Gamma_0. \end{cases}$$

For $m, n \in \mathbf{N}$, m odd, $u_{m,n} = \cos\left(\frac{m\pi}{2}(1-x)\right)\cos(n\pi y)$ verifies

$$\begin{cases} -\triangle u_{m,n} = \lambda_{m,n} u_{m,n} & \text{in} \quad \Omega, \\ \frac{\partial}{\partial n} u_{m,n} = 0 \quad \text{on} \quad \Gamma, \\ u_{m,n} = 0 \quad \text{on} \quad \Gamma_0, \end{cases}$$

where $\lambda_{m,n} = \left(\frac{m^2}{4} + n^2\right)\pi^2$. A direct calculation gives

$$\phi_t = \sum_{m,n} \phi'_{m,n} u_{m,n},$$

where

$$\phi'_{m,n} = 4(\varphi_t, u_{m,n})_{L^2(\Omega)} = \frac{2p'_n}{\cosh(n\pi)} \int_0^1 \cosh(n\pi x)\cos\left(\frac{m\pi}{2}x\right)dx = \frac{m\pi p'_n}{\lambda_{m,n}} \sin\left(\frac{m\pi}{2}\right). \tag{3.3}$$

By Duhamel's formula, we deduce that

$$Q = -\sum_{m,n} \sin\left(m\frac{\pi}{2}\right)\frac{m\pi}{\lambda_{m,n}} u_{m,n} \int_0^t e^{-\lambda_{m,n}k(t-s)} p'_n(s)\, ds.$$

Defining $\gamma(P) = \frac{\partial P}{\partial n}_{|\Gamma_0}$, we get for $t > 0$

$$\gamma(Q)(t) = \sum_{m,n} \frac{m^2\pi^2}{2\lambda_{m,n}} \cos(n\pi y) \int_0^t e^{-\lambda_{m,n}k(t-s)} p'_n(s)\, ds,$$

and

$$\gamma(\phi)(t) = -\sum_n p_n(t) n\pi \tanh(n\pi)\cos(n\pi y).$$

In particular, since $P = \phi + Q$,

$$\begin{aligned} \textstyle\int_{\Gamma_0} \gamma(P)(t) &= \textstyle\int_{\Gamma_0} \gamma(Q)(t) + \int_{\Gamma_0} \gamma(\phi)(t) = \int_{\Gamma_0} \gamma(Q)(t) \\ &= \textstyle\sum_m \frac{m^2\pi^2}{2\lambda_{m,0}} \int_0^t e^{-\lambda_{m,0}k(t-s)} p'_0(s)\, ds. \end{aligned}$$

Since $\lambda_{m,0} = m^2\pi^2/4$, we obtain

$$\begin{aligned} k \textstyle\int_{\Gamma_0} \gamma(P)(t) &= 2k \textstyle\sum_m \int_0^t e^{-m^2\pi^2 k(t-s)/4} p'_0(s)\, ds \\ &= \textstyle\frac{8}{\pi^2} \int_0^t \frac{1}{t-s} f(\pi^2 k(t-s)/4) p'_0(s)\, ds, \end{aligned} \tag{3.4}$$

where

$$f(\sigma) = \sigma \sum_m e^{-m^2\sigma}. \tag{3.5}$$

Note that $f'(\sigma) = \sum_m (1-m^2\sigma)e^{-m^2\sigma}$. Setting $h = 2\sqrt{\sigma}$ and $F(x) = (1-x^2)e^{-x^2}$, we have

$$2\sqrt{\sigma} f'(\sigma) = \sum_{i=0}^{\infty} hF\left(ih + \frac{h}{2}\right) \underset{h\downarrow 0}{\longrightarrow} \int_0^{\infty} F(x)\,dx;$$

and so,

$$2\sqrt{\sigma} f'(\sigma) \underset{\sigma\downarrow 0}{\longrightarrow} \int_0^{\infty} F(x)\,dx = \frac{1}{2}\int_0^{\infty} e^{-x^2}\,dx > 0, \tag{3.6}$$

where the last identity follows by integration by parts. We deduce in particular from (3.6) that $f(\sigma)$ is (strictly) increasing on $(0, \sigma_0)$ for some $\sigma_0 > 0$.

Suppose now $p_{k_1} = p_{k_2} = p$, and assume by contradiction that

$$k_1 < k_2.$$

Then, from the second equation of (2.1) we have $v_{k_1} = v_{k_2}$ and thus, using the first equation,

$$k_1\gamma(P_{k_1}) = k_2\gamma(P_{k_2}); \tag{3.7}$$

and it follows from (3.4) that

$$\int_0^t \frac{1}{t-s}\Big(f(\pi^2 k_1(t-s)/4) - f(\pi^2 k_2(t-s)/4)\Big)\,p_0'(s)\,ds = 0. \tag{3.8}$$

By (3.1), $p_0' > 0$ on $[0, \tau]$ for some $\tau > 0$, and since f is increasing on the interval $(0, \sigma_0)$, it follows that the left-hand side of (3.8) is negative for t small enough, which yields a contradiction.

□

Remark 3.2. The permeability k is also univocally determined if $U_0 = (p_0, v_0, P_0) \in D(A)$, $\alpha \in C^2([0,T])$ and $\gamma(P_0) \neq 0$. In fact, suppose $p_{k_1} = p_{k_2}$ over $\Gamma_0 \times [0,T]$. Then (3.7) still holds over $[0,T]$ and yields $k_1\gamma(P_0) = k_2\gamma(P_0)$, which implies $k_1 = k_2$.

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